

Compare Linear and Nonlinear Functions

FOCUS

Mathematics Objective

- Use different representations to compare linear and nonlinear functions.

Language Objective Explain, orally and in writing, how to compare linear and nonlinear functions.

Essential Understanding Two functions presented in different representations can be compared by looking at their properties: initial value and constant rate of change.

COHERENCE

In Grade 7, students:

- analyzed and used proportional relationships.
- solved problems using equations and inequalities.
- graphed solutions of equations and inequalities.

In this lesson, students:

- compare properties of two linear functions.
- compare a linear and nonlinear function.

Later in this topic, students will:

- construct functions to model linear relationships.
- analyze intervals of increase and decrease.
- sketch functions from verbal descriptions.

Additional Connections Graphing functions connects to graphing linear equations.

RIGOR

Conceptual Understanding Students use representations, such as equations, tables, and graphs, to compare the properties of linear and nonlinear functions.

Application Students apply their knowledge of functions to solve real-world problems using equations, tables, and graphs.

Vocabulary

Revisit previously taught vocabulary

Materials

- graph paper

Student Resources

 Family Engagement

Intervention System (IS)

K52 Linear Functions

Teacher Resources

Available at
Savvas Realize®.

 Editable Lesson Plan

 Lesson Presentation Slides

 enVision on the Go: Planning Support



Notes

Language Support

Use this activity to support understanding of the terms *linear function* and *nonlinear function* with Example 2.

Questions that Assess

Understanding Read aloud the sentences below the table and graph. Point out the terms *linear function* and *nonlinear function*. Ask:

- What does *linear* mean? [straight line]
- What does *nonlinear* mean? [not in a straight line]
- How does knowing these terms help you define what a *linear function* and a *nonlinear function* mean?

On the left half of a sheet of paper, have students write the term *linear function* and describe its characteristics. On the right side, have them write the term *nonlinear function* and describe its characteristics.

Invite students to read their descriptions aloud to the class. Encourage students listening to provide feedback by suggesting characteristics that might be added to, revised, or deleted from the descriptions.

An additional activity is available online.

- Use the activity that supports understanding of the term *constant* with Example 1.

Math Talk

Strategize First Steps

Decide on a first step.

Marcus went for a hike in the woods. He made a couple of notes along the way. What was his average walking rate?

Time (hours)	$\frac{1}{4}$	$1\frac{1}{2}$
Distance (miles)	0.8	4.8

Problem Solving Students strategize about properties of linear functions as they consider possible first steps for this problem. Use this Math Talk at the beginning of the lesson.

Tell students that the focus of the discussion should be on the first step, and not on solving the problem.

As students share, support them in making connections among the strategies. Ask:

- When would you find it helpful to graph the points?
- When do you need to find the slope? the y-intercept? Which would you find first?
- How does graphing the points help you find the slope? the y-intercept?

At this point in their learning, students are likely to present several possible first steps.

Exit Ticket

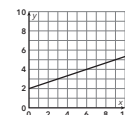


Name **Use this Exit Ticket anytime after Example 1.**

Exit Ticket
Lesson 3-3

1. Compare the properties of the two functions.

Function A



Function B

x	y
1	2
3	8
5	14
7	20

Which statement is true?

- Ⓐ Function A has a greater rate of change than Function B.
- Ⓑ Function B has a greater rate of change than Function A.
- Ⓒ Function B has a greater initial value than Function A.
- Ⓓ Function A is a linear function, but function B is not.

Self-Assessment

Select how well you understand what you learned in this lesson.

Math Goal

I can compare linear and nonlinear functions.

Got it

Need help

Working on it

Language Goal

I can explain, orally and in writing, how to compare linear and nonlinear functions.

Got it

Need help

Working on it

Teacher Answer Key

1 of 1

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You can use the Assessment Sourcebook version or the auto-scored version on Savvas Realize®.

Prevent Misconceptions

When students work on Do You Understand Item 2, they may incorrectly determine the rate of change for each linear function. Remind them that the rate of change is the change in the y-values divided by the change in the x-values.

Use sentence frames to determine the rate of change for each function before comparing them.

- The y-values change by _____ as the x-values change by _____.
- This can be written as the ratio _____.

English Language Development

ELL English Language Learners (Example 1)

MLR3 Critique, Correct, and Clarify (Example 3)

Language Support (Example 2)

Step 1 Explore and Share



15–20 min

Explore and Share Scout Observational Assessment

Compare Linear and Nonlinear Functions

I can ... compare linear and nonlinear functions.

Let's Build Lesson 3-3

Explore and Share

Two media streaming services offer family plans with different monthly costs, as shown in the ads. What do the two plans have in common? How are they different? When is Media4You a better deal than Family Stream?

Model with Math How can you represent the relationship between cost and number of users?

Look for Relationships Describe the relationship between the cost and the number of users for each service. What do you notice about each relationship?

Sample answer: The relationship is linear for Media4You because there is a constant rate of change of \$2 per use after the first one. The relationship is not linear for Family Stream because the rate of change in cost with respect to number of users is not constant.

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Sample Student Work

Media4You

Number of Devices	1	2	3	4	5	6
Cost	\$10	\$12	\$14	\$16	\$18	\$20

Family Stream

Number of Devices	1	2	3	4	5	6
Cost	\$12	\$12	\$12	\$12	\$13	\$14

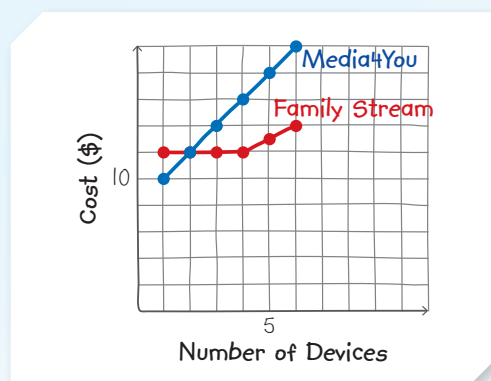
Media4You is a better deal when you have 1 device.

Zachary's Work

Zachary uses tables to determine when the cost is lower for the Media4You subscription than for Family Stream.

Elizabeth's Work

Elizabeth uses digital drawing tools to graph the cost of different numbers of devices for each service.



ETP Explore and Share



Purpose Students compare a linear and nonlinear function to support comparing linear functions by their rates of change in Example 2.

Introduce WHOLE CLASS

Ask: Does your family subscribe to a streaming service? Does it cost more for the service to be used on more than one device?

Monitor, Select, and Sequence SMALL GROUPS

Observe students at work.

- What strategies do students use to organize the data about the streaming services? Students may create one or two tables of values showing the cost of each plan given different numbers of devices.

Students then compare their representations to find the point where Media4You costs less than Family Stream. If needed, ask: What representations would you use to model the two services?

Early Finishers

Ask: How would the problem change if the first device cost \$5 for Media4You? [Media4You would be a better deal for up to 4 devices, and cost the same as Family Stream for 5 devices.]

Connect WHOLE CLASS

Connect Solution Strategies Have students present their solutions. Consider having students who used a table present before those who used a graph. Have them discuss how the values in the table are represented in the graph; the pairs of values for the number of devices and the cost are the coordinate pairs in the graph.

Have students discuss how the number of allowable devices affects the cost of each plan. Does it grow constantly, or does it vary? For Media4You, the cost grows constantly with the number of devices. For Family Stream, it does not grow at all until the fifth device, then grows constantly. Have students discuss whether they think Media4You will ever become the better deal again.

Connect to the Visual Learning When presenting Example 2, discuss how the behavior of the rates of change differ between the two functions in the Explore and Share; the function that represents Media4You has a constant rate of change, \$2 per device, so it is a linear function. The function that represents Family Stream does not have a constant rate of change; it is a nonlinear function.

Visual Learning Animation

Essential Question

How can you compare two functions?

Visual Learning

Example 1 Compare Two Linear Functions

An auto assembly factory needs to purchase new welding robots. The factory manager has information on two different models of welding robots. The welding rates for each model are shown. How do the welding rates for the two robots compare?



Look for Relationships What properties of functions can be used to compare functions?

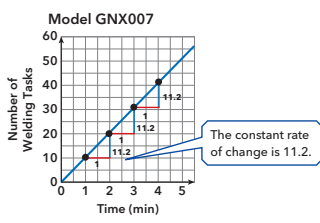
STEP 1

Find the welding rate, or the constant rate of change, for each robot.

Model T1000

Time (minutes)	Number of Welding Tasks
2	20.8
5	52
7	72.8
12	124.8

Model GNX007



The constant rate of change is 11.2.

$\frac{52 - 20.8}{5 - 2} = \frac{31.2}{3} = 10.4$

The constant rate of change is 10.4.

STEP 2

Find the initial value for each robot.

At 0 minutes, each robot has performed 0 tasks, so the initial value is 0.

The data for the model GNX007 robot has a greater constant rate of change, or welding rate, so it can complete more welding tasks per minute than the Model T1000 robot.

Try It!

The welding rate of a third robot is represented by the equation $t = 10.8w$, where t represents the time in minutes and w represents the number of welding tasks. How does it compare to the other two?

The welding rate, or constant rate of change, is 10.8, which is greater than the constant rate of change for the Model T1000 robot, but less than that for the Model GNX007 robot. It is faster than the Model T1000 robot but slower than the Model GNX007 robot.

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ETP Essential Question

Engage students in a discussion about the **Essential Question**. Make sure students understand that there are different representations and properties of functions that can be used to make comparisons.

ETP Example 1 Compare Two Linear Functions

- What properties can be used to compare two functions? [You can use the initial value and the rate of change.]
- What does the constant rate of change for each model mean? [The number of tasks a robot can complete per minute.]

ETP Try It!

- How can you use the equation $y = 10.8w$ to compare the third function to the other two? [Compare the value of m in the equation $y = 10.8w$ to the constant rates of change of the other two functions.]
- What is another way you can compare linear functions using an equation? [You can use an equation to identify the initial value, or y -intercept.]

Support Student Understanding

Example 1 Some students may need to review how to identify the constant of proportionality and represent proportional relationships using equations.

- What is the unit rate for this situation: *Sidney purchased three video games for \$45.75. Each game cost the same. What was the cost for one video game?* [\$15.25; $45.75 \div 3 = 15.25$]
- Write an equation to represent the total cost for the number of movie tickets purchased, where c represents the total cost, t represents the number of movie tickets purchased, and each movie ticket costs \$9.75. What is the constant of proportionality? [$c = 9.75t$; 9.75]

Extend Student Thinking

Example 2 Challenge students to compare the two relationships using equations and graphs.

- What equation models the linear function represented in the table? [$y = 4x$, where x represents the side length and y represents the perimeter of the square]
- What equation models the nonlinear function represented by the graph? [$y = x^2$, where x represents the side length and y represents the area of the square]
- Graph the relationship represented in the table. How does this differ from the graph showing the relationship between the side lengths and area? [Check students' graphs. The graph of the relationship represented in the table is a straight line and is a linear function. The graph shown is not a straight line and is a nonlinear function.]

Step 2 Visual Learning *continued*

Example 2 Compare a Linear and a Nonlinear Function

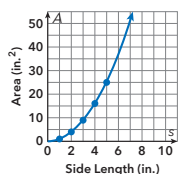
A square with side length s is shown. The table shows the relationship between the side length and perimeter as the side length increases. The graph shows the relationship between the side length and area. How do the two relationships compare?



Side Length, s (in.)	Perimeter, P (in.)
0	0
1	4
2	8
3	12
4	16

This relation is a function. It has a constant rate of change. It is a *linear* function.

Both relationships are functions. Both perimeter and area are functions of side length.



This relation is a function, but it does not have a constant rate of change. It is a *nonlinear* function.

Example 3 Compare Properties of Linear Functions

Compare the properties of the two functions.

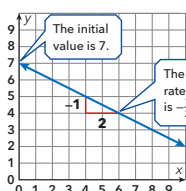
Function A

The constant rate of change is 2.

$$y = 2x - 3$$

The initial value is -3 .

Function B



The initial value is 7.

The constant rate of change is $-\frac{3}{2}$.

Function A has a greater rate of change. Function B has a greater initial value.

Try It! Compare the properties of these two linear functions.

Function 1

x	2	5	9	11	14
y	1	5.5	11.5	14.5	19

Function 2

$$y = 2x - 4$$

Function 2 has a greater rate of change.
Function 1 has a greater initial value.

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ETP Example 2 Compare a Linear and a Nonlinear Function

- How can you determine whether the relationship in the table is a function? [Check to see that each input value (length) has exactly one output value (perimeter).]
- What are the similarities and differences between the two relationships? [Both relationships are functions; one is linear, one is nonlinear.]

ETP Example 3 Compare Properties of Linear Functions

- Which properties are being compared? [The initial value and constant rate of change]
- What does the initial value of a function equate to in a linear equation? [The y -intercept, or b]
- What does the constant rate of change of a function equate to in a linear equation? [The slope, or m]

ETP Try It!

- What two properties can you compare? [Rate of change and initial value]
- What are the rates of change for each function? [The rate of change for Function 1 is 1.5; for Function 2 is 2.]
- What is the initial value for each function? [The initial value for Function 1 is -2 ; for Function 2 is -4 .]

MLR3 Critique, Correct, and Clarify

Example 3 Try It! Have partners work together to develop an explanation of how to determine the slope and initial value of a function from an equation, table, and graph. Then have pairs share their explanations. Encourage students to offer suggestions for how to enhance the explanations.

ELL English Language Learners SPEAKING

Emerging Support students' understanding of the non-math vocabulary in Example 1.

Have students identify the robots.

- What do you think these robots do?

Help students understand that these are welding robots that weld metal together.

Ask what factories produce.

- What kinds of products are made in factories?

Expanding Have students discuss different kinds of robots that they may know about.

- What kinds of robots can you think of? [Robots in factories to help make different kinds of goods—electronics, cars, appliances]
- Why do businesses use robots? [They are more efficient than people. They can work without taking a break.]

Bridging Have students investigate other tasks that robots can do besides welding.

- What other jobs can robots do? [Painting, assembly]
- Are robots good for businesses? Are they good for people? [They are good for businesses, but not good for some workers.]

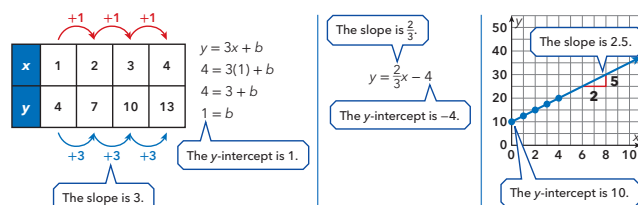
Have students discuss their views on robots.

Step 2 Visual Learning *continued*

Key Concept

You can compare functions in different representations by using the properties of functions.

Compare the constant rate of change and the initial value.



Talk About Math Ideas

How do different representations of functions help you determine whether a function is linear or non-linear?

Do You Understand?

- Essential Question** How can you compare two functions?

Sample answer: You can compare two functions by looking at the properties of the functions, such as initial value and rate of change.

- Reasoning** Anne is running on a trail at an average speed of 6 miles per hour beginning at mile marker 4. John is running on the same trail at a constant speed, shown in the table. Who is running faster? Explain.

Time (hours), x	0	0.5	1	1.5
Mile Marker, y	1	4.5	8	11.5

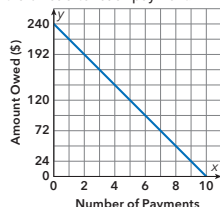
Sample answer: John; Anne's speed is 6 mi/h. Using two ordered pairs from the table, John's speed is 7 mi/h.

- Reasoning** In Item 2, how do Anne and John's starting positions compare? Explain.

Sample answer: Anne starts at mile marker 4. John starts at mile marker 1.

Do You Know How?

Felipe and Samantha use a payment plan to buy musical instruments. Felipe writes the equation $y = -30x + 290$ to represent the amount he owes, y , after x payments. The graph shows how much Samantha owes after each payment.



- Whose musical instrument costs more? Explain.
Felipe's; Sample answer: Look at the initial values. Felipe's costs \$290 and Samantha's costs \$240.
- Who will pay more each month? Explain.
Felipe; Sample answer: Look at the constant rates of change. He will pay \$30 each month and Samantha will pay \$24.

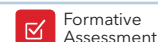
ETP Key Concept

- Why is the graph of a linear function a straight line and a nonlinear function not? [The graph of a linear function has a constant rate of change and the graph of a nonlinear function does not have a constant rate of change.]

ETP Talk About Math Ideas

Have students engage in a think-pair-share to discuss the response to the question comparing linear and nonlinear functions. Listen for discussion of how a constant rate of change indicates a linear function, while a nonlinear function does not have a constant rate of change.

ETP Do You Understand/Do You Know How?



Essential Question

Students should understand how to compare two functions in different representations by looking at their properties.

Item 1

- How can you determine the constant rates of change of relationships presented in tables or as equations? [For data in a table, divide the difference of two y-values by the difference of the corresponding x-values. In an equation, look for the value of m .]

Prevent Misconceptions

Item 4 If students begin the problem with the incorrect starting amounts borrowed for the instruments, remind them that the starting amount is the initial value, or the y-value when x is 0.

- What is the value of y when x is 0 in the equation $y = -30x + 290$? Show your work.
[290; $y = -30(0) + 290 = 0 + 290 = 290$]
- What is the value of y when x is 0 on the graph? [240]

Step 3 Practice and Problem Solving



15-20 min

Interactive Practice Adaptive Practice

You may opt to have students complete automatically scored practice items online.

Practice and Problem Solving

6. Two linear functions are shown. Which function has the greater rate of change?

Function A

Function B

x	y
0	0
2	3
4	6
6	9

Function B

x	y
-1	6
0	4
1	2
2	0

Function A

7. Two linear functions are shown. Which function has the greater initial value?

Function A

x	y
-1	6
0	4
1	2
2	0

Function B

Function B

$y = 7x + 3$

8. Is each function linear or nonlinear?

Function A

x	y
0	1
1	2
2	5
3	10

Function B

Function A is nonlinear.
Function B is linear.

9. Is each function linear or nonlinear?

Function A

Function B

Function B

$y = x$

Function A is nonlinear.
Function B is linear.

10. Determine whether each function is linear or nonlinear from its graph.

Function I

Function II

Function I is linear.
Function II is nonlinear.

11. Look for Relationships Justin opens a savings account with \$4. He saves \$2 each week. Does a linear function or a nonlinear function represent this situation? Explain.

Week	0	1	2	3	4	5
Money in Account	4	6	8	10	12	14

linear; There is a constant rate of change.

12. Reasoning The function $y = 4x + 3$ describes Player A's scores in a game of trivia, where x is the number of questions answered correctly and y is the score. The function represented in the table shows Player B's scores. What do the rates of change tell you about how each player earns points?

Sample answer: Player A earns more points than Player B for each additional correct answer.

Correct Answers	Score
1	4
2	5
3	6
4	7

13. Two athletes are training over a two-week period to increase the number of push-ups each can do consecutively. Athlete A can do 16 push-ups to start, and increases his total by 2 each day. Athlete B's progress is charted in the table. Compare the initial values for each. What does the initial value mean in this situation?

Sample answer: The initial value for Athlete A is greater. This means that Athlete A was able to do more push-ups than Athlete B when the training started.

Day	Number of Push-ups
0	12
1	15
2	18
2	21

14. Higher Order Thinking The equation $y = 4x - 2$ and the table and graph shown describe three different linear functions. Which function has the greatest rate of change? Which has the least? Explain.

The function in the table has the greatest rate of change; The function in the graph has the least rate of change; Sample answer: The slope of the function in the table is 5, the slope of the equation is 4, and the slope of the graph is -2.

x	y
1	5
2	10
3	15
4	20

Assessment Practice

15. Elijah has 2 cats and 2 dogs. He finds that he uses 3 cups of dog food to feed his dogs each day. After 12 days he has about 84 cups of dog food left. The table shows the amount of cat food remaining from feeding the cats after different amounts of time. Which animals consumed their food at a faster rate?

Dogs; the rate for the cats is 2 cups per day; the rate for the dogs is 3 cups per day.

Time Elapsed (days)	Cups of Cat Food
6	48
8	44
12	36
14	32

16. The height of a burning candle can be modeled by a linear function. Candle A has an initial height of 201 millimeters, and its height decreases to 177 millimeters after 4 hours of burning. The height, h , in millimeters, of Candle B can be modeled by the function $h = 290 - 5t$, where t is the time in hours. Which of the following statements are true?

- ☐ The initial height of Candle A is greater than the initial height of Candle B.
- ☒ The height of Candle A decreases at a faster rate than the height of Candle B.
- ☒ Candle B will burn out in 58 hours.
- ☐ After 10 hours, the height of Candle A is 110 millimeters.
- ☒ Candle A will burn out before Candle B.

Practice and Problem Solving

Error Intervention

Item 12 Reasoning If students say that Player B gains more points than Player A, review the definition of rate of change.

- What is the rate of change for Player A? Player B? [4; 1]
- What does the rate of change mean in each situation? [For each correct answer, Player A scores 4 points and Player B scores 1 point.]

Challenge

Item 14 Higher Order Thinking You can use this item to extend students' understanding of slope, or rate of change.

- For which representation can you determine the slope most easily? [The equation in the form $y = mx + b$; look for the value of m .]

Engage Through Student Choice

Go online for an assignment strategy.

Item Analysis: Practice and Problem Solving

Example	Items	DOK
1	6	2
	14-15	3
2	8-11	2
3	7, 13, 16	2
	12	3

Step 4 Assess and Differentiate



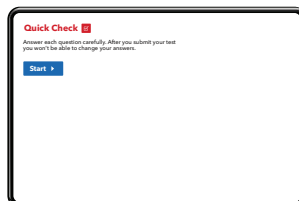
15-30 min

ETP Quick Check



Assess lesson content with the Quick Check.

- The online Quick Check is **automatically scored** and **remediation, practice, or enrichment** will be assigned based on student performance.
- You can use the Assessment Sourcebook version to prescribe targeted remediation or enrichment.



0-3 points: Reteach to Build Understanding and Additional Vocabulary Support Activities

4-5 points: Enrichment Activity

In addition, **English Language Learners** may benefit from the Build Mathematical Literacy Activity.

Differentiation Library DL

- Additional Differentiation Library resources are described at the start of this topic.
- Differentiation resources that support diverse learning styles are online and in the Differentiation Library Teacher's Guide.

Video Tutorials Available on Savvas Realize®

Students can access instructional tutorials online.

- How Do You Compare Integers Using a Number Line?
- How Do You Represent Real-World Situations Using Integers?

Item Analysis: Additional Practice

Example	Items	DOK
1	1, 2, 5	2
2	3	1
	4	2
3	7-9	2
	6	3

Additional Practice

Printable or autoscored on Savvas Realize®

3-3 Additional Practice

1. Two linear functions are shown below. Which function has the greater rate of change?

Function A

x	y
4	32
8	44
12	56
16	68
20	80

Function A

Function B

2. Two linear functions are shown below. Which function has the greater initial value?

Function A

 $y = \frac{3}{2}x + 5$

Function B

x	-2	0	2	4	6
y	5	8	11	14	17

3. Tell whether each function is linear or nonlinear.

Function A

x	-2	1	2	3	4
y	3	0	-1	-2	-3

Function A is linear;
Function B is nonlinear

Function B

4. Decide whether each function is linear or nonlinear from its graph.

Function I

nonlinear

Function II

linear

Function III

nonlinear

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5. **Look for Relationships** Which two functions have the same rate of change?

A. $y = 0.5x - 1$ B. $y = 4x - 7$ C. $n = 0.6r + 1$ D. $t = 0.5n + 1$

Functions A and D have the same rate of change.

6. **Make Sense and Persevere** Glen compares the rates of change of two linear functions represented in different forms.

a. For a linear function in the form $y = mx + b$, how does Glen determine the rate of change?
Look at the value of m.

b. How can Glen determine the constant rate of change of the linear function presented in the table on the right? What is the rate of change?
Sample answer: Find the change in y divided by the change in x. The rate of change is -2.

x	y
1	22
2	20
3	18
4	16

7. **Critique Reasoning** The table on the right and the equation $y = 8x + 5$ describe linear functions. A student states incorrectly that the initial values of the functions are equal. Compare the initial values of the functions. What mistake did the student likely make?
The initial value in the table is greater. The student likely compared the rate of change for the function in equation form to the initial value for the function in table form.

x	y
0	8
1	9
2	10
3	11

Assessment Practice

8. The equation $y = 4x + 60$ and the table each describe a linear function. Compare the properties of the functions. Select all that apply.

x	10	20	30	40
y	60	80	100	120

☐ The linear function described by the table has the greater rate of change.

☒ The linear function described by the equation has the greater rate of change.

☐ The rates of change are equal.

☐ The linear function described by the table has the greater initial value.

☒ The linear function described by the equation has the greater initial value.

☐ The initial values are equal.

9. Jeff saved \$500 from his summer job so he would have spending money during the school year. He withdraws \$12 from his account each week, so a linear function models his plan. Melissa made a similar plan. The table shows the results of her first five transactions. Compare the functions.

Melissa's Savings

Week	1	2	3	4	5
Balance	\$510	\$500	\$490	\$480	\$470

Sample answer: Jeff withdraws more money from his account each week, so his function has the greater rate of change. Melissa's function has the greater initial value because she had \$10 more at the start than Jeff.

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